22nd order high-temperature expansion of nearest-neighbor models with O(2) symmetry on a simple cubic lattice.

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Abstract

We present the high-temperature series for a nearest-neighbor model with O(2) symmetry on a simple cubic lattice with the most general single-site potential. In particular, the magnetic susceptibility and the second-moment correlation length are computed to 22nd order. The series specialized to some particular improved Hamiltonians have been already analyzed in the paper M. Campostrini, M. Hasenbusch, A. Pelissetto, and E. Vicari, Phys. Rev. B 74, 144506 (2006) [cond-mat/0605083], to determine the critical exponents and other universal quantities of the three-dimensional XY universality class.

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We have computed the high-temperature series for O(2)-symmetric nearest-neighbor models on a simple cubic lattice with the most general single-site potential. We consider two-dimensional real vectors $\vec{\phi}_x$ defined at the sites x of the lattice, the nearest-neighbor Hamiltonian

$$\mathcal{H} = -\beta \sum_{\langle xy \rangle} \vec{\phi}_x \cdot \vec{\phi}_y \tag{1}$$

and the partition function

$$Z = \int \prod_x d\mu(\phi_x)\, e^{-\mathcal{H}}. \tag{2}$$

Different models correspond to different choices of the single-site measure $d\mu(\phi_x)$. Three different models have been considered in the paper 1:

(i) XY model:

$$d\mu(\phi_x) = d\phi_x^{(1)}\, d\phi_x^{(2)}\, \delta(1 - |\phi_x|)\ ; \tag{3}$$

(ii) ddXY model:

$$d\mu(\phi_x) = d\phi_x^{(1)}\, \phi_x^{(2)}\, e^{D|\phi_x|^2} \left[\delta(\phi_x^{(1)})\, \delta(\phi_x^{(2)}) + \frac{1}{2\pi}\, \delta(1 - |\phi_x|) \right] ; \tag{4}$$

(iii) ϕ^4 lattice model:

$$d\mu(\phi_x) = d\phi_x^{(1)}\, d\phi_x^{(2)}\, \exp\left[-\vec{\phi}_x^2 - \lambda(\vec{\phi}_x^2 - 1)^2 \right]. \tag{5}$$

Using the linked-cluster expansion technique, we have computed the high-temperature expansion of several quantities. We considered the magnetic susceptibility χ

$$\chi = \sum_x \langle \phi_\alpha(0) \phi_\alpha(x) \rangle, \tag{6}$$

and the zero-momentum connected $2j$-point Green's functions χ_{2j} ($\chi = \chi_2$) for $j = 2, 3, 4, 5$:

$$\chi_{2j} = \sum_{x_2, \ldots, x_{2j}} \langle \phi_{\alpha_1}(0) \phi_{\alpha_1}(x_2) \ldots \phi_{\alpha_j}(x_{2j-1}) \phi_{\alpha_j}(x_{2j}) \rangle_c. \tag{7}$$

We also considered the first moments of the two-point function

$$m_{2k} = \sum_x x^{2k} \langle \phi_\alpha(0) \phi_\alpha(x) \rangle. \tag{8}$$

More precisely, we computed χ to 22nd order, χ_4 to 20th order, χ_6 and χ_8 to 18th order, χ_{10} to 15th order, m_2 to 22nd order, m_4 to 20th order, m_6 and m_8 to 19th order.

The high-temperature series specialized to some particular cases, providing Hamiltonians with suppressed scaling corrections, have been already analyzed in Refs. 1,2.

The quantities necessary to reconstruct the series are reported in the TXT files `c_phi4.TXT` and `series.TXT` which can be found in Ref. 3.

For each quantity the reported high-temperature series is written as

$$O = \sum_{k,i_1,\ldots,i_{13}} \beta^k c_1^{i_1} \ldots c_{13}^{i_{13}} O(k; i_1, \ldots, i_{13}) \ , \tag{9}$$

with coefficients c_k defined below. The presence of c_k only for $k \leq 13$ is of course related to the length of the present series: longer series require additional coefficients c_k. Indeed, in order to compute the n-th term in the expansion of χ_l, one needs c_k with $k \leq \lfloor (n+l)/2 \rfloor$. For m_l, we have analogously $k \leq \lfloor (n+1)/2 \rfloor$, independent of l.

The coefficients c_k depend on the single-site measure and are defined as

$$c_k = \frac{1}{2^k k!} \frac{I_{1+2k}}{I_1} \qquad\qquad I_k = \int_0^\infty x^k f(x), \tag{10}$$

where $f(x)$ is defined by the single-site measure:

$$d\mu(\phi_x) = d\phi_x^{(1)}\, d\phi_x^{(2)}\, f(|\phi_x|). \tag{11}$$

In file `c_phi4.TXT` of Ref. 3 we report the quantities c_k for $k = 1, 13$ for the ϕ_4 theory with $1.90 \leq \lambda \leq 2.20$. Each line contains three numbers: λ, k, c_k. For the ddXY model, the coefficients c_k can be computed analytically:

$$c_k = \frac{1}{2^k k!} \left[\delta_{k0} + (1 - \delta_{k0}) \frac{1}{1 + e^{-D}} \right] \ . \tag{12}$$

For the XY model

$$c_k = \frac{1}{2^k k!} \ . \tag{13}$$

The high-temperature series are reported in file `series.TXT` of Ref. 3. Each line contains 16 numbers: the first number indicates which observable one is referring to: 1 refers to χ, 2 to χ_4, 3 to χ_6, 4 to χ_8, 5 to χ_{10}, 6 to m_2, 7 to m_4, 8 to m_6, 9 to m_8. The following 14 numbers are respectively k, $i_1, \ldots, i_{13}$ in Eq. (9). The last number is the coefficient $O(k; i_1, \ldots, i_{13})$.

We only report the nonvanishing coefficients.

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[1] M. Campostrini, M. Hasenbusch, A. Pelissetto, and E. Vicari, *Theoretical estimates of the critical exponents of the superfluid transition in ^{4}He in lattice methods*, Phys. Rev. B **74**, 144506 (2006) [cond-mat/0605083].

[2] M. Campostrini, M. Hasenbusch, A. Pelissetto, P. Rossi, E. Vicari, *Critical behavior of the XY universality class*, Phys. Rev. B **63**, 214503 (2001) [cond-mat/0010360].

[3] EPAPS Document No. E-PRBMDO-74-051634, which can be reached via a direct link in the HTML reference section of the online version of the article 1 or via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html).